Ques 1:

a). Model was trained on data from stable economic conditions and tested on data from economic downturn. Training data distribution no longer matches testing distribution.

b). High quality, diverse and representative data enables the model to capture general patterns. A simple model can generalize well with good data, while a complex model will fail with biased data.

c). Large volume of data is normal and unlabeled. Very few labeled fraud cases exist. Clustering based anomaly detection algorithm can be used.

d). Outcome is binary with a probability p of success. Bernoulli distribution can be used.

Ques 2:

a). Initial Parameters: $w = 0.1$, $b = 0$, $\alpha : 0.05$

Applicant Data:

Applicant	Income (x)	Default (y)
A	$2 \rightarrow x^1$	$0 \rightarrow y^1$
B	$5 \rightarrow x^2$	$1 \rightarrow y^2$

Predicted Probability:

Applicant A,

$$z^1 = wx^1 + b = 0.1 \times 2 + 0 = 0.2$$

$$\sigma(z^1) = \frac{1}{1 + e^{-z^1}} = \frac{1}{1 + e^{-0.2}} = \frac{1}{1 + 0.8187} = 0.5498$$

Applicant B,

$$z^2 = wx^2 + b = 0.1 \times 5 + 0 = 0.5$$

$$\sigma(z^2) = \frac{1}{1 + e^{-z^2}} = \frac{1}{1 + e^{-0.5}} = \frac{1}{1 + 6065} = 0.6225$$

$$\hat{y}^1 = 0.55 \qquad \hat{y}^2 = 0.62$$

Batch Gradient:

$$\frac{\partial L}{\partial w} = \frac{-1}{m}\left(\sum_{i=1}^{m} y^{(i)} - \hat{y}^{(i)}\right)x^{(i)}$$

$$= \frac{1}{m}\left(\sum_{i=1}^{m} \hat{y}^{(i)} - y^{(i)}\right)x^{(i)}$$

Applicant A, $(\hat{y}^1 - y^1)x^1 = (0.5498 - 0)(2) = 1.0996$

Applicant B, $(\hat{y}^2 - y^2)x^2 = (0.6225 - 1)(5) = -1.8875$

$$\frac{\partial L}{\partial w} = \frac{1}{2}(1.0996 - 1.8875) = \frac{-0.7879}{2} = -0.394$$

$$\frac{\partial L}{\partial w} = -0.394$$

b). SVM decision boundary $f(x) = 0.8\,x_1 - 1.5\,x_2 - 50 = 0$

Substitute new Stock values:

$$f(x) = 0.8 * 72 - 1.5 * 15 - 50$$
$$= -14.9$$

Buy class support vector $f(x) = 0.8 * 75 - 1.5 * 10 - 50$
$= -5$

Sell class support Vector $f(x) = 0.8 * 60 - 1.5 * 20 - 50$
$= -32$

Importance of Support vectors:

SV are data points closest to decision boundary. They are the critical points that constrain the optimal separating hyperplane.

Why removing non-support vectors doesnot affect the boundary:

Non-support vectors lie outside the margin. They donot contribute to the optimization constraints.

c). Post-pruning is better giving validation accuracy of 91%.

Ques 3:

a).

Avg Intra Distance measures how compact the cluster is. (lower is better)

Closest Inter Distance measures how well the cluster is separated from other clusters (higher is better).

Among C1, C2 and C3, C2 has highest intra distance and lowest inter distance. C2 shows the strongest indication of cluster assignment ambiguity.

b). Cumulative Variance:

PC1 + PC2 : 58 + 30 = 88%

PC1 + PC2 + PC3 : 58 + 30 + 9 = 97%

PC1 + PC2 + PC3 + PC4 : 58 + 30 + 9 + 3 = 100%

c).

Precision: among all predicted as +ve, how many are actually +ve.

Recall: among those which are actually +ve, how many were predicted +ve.

Security team goal is to increase Recall
and
decrease unnecessary alerts

→ decrease false Positives → Increase Precision

Threshold	Precision	Recall
0.40 0.75	0.72 0.93	0.88 0.46

→ 88% Recall and 28% FP

→ 46% Recall and 7% FP

Threshold 0.4 preferrable

Ques 4:

a). Validation accuracy represents generalization to unseen data.

Method	Batch Size	Training Accuracy	Validation Accuracy
Full Batch GD	120,000	72%	70%
Mini Batch GD	256	89%	87%
Stochastic GD	1	97%	75%

Mini Batch GD performs best.

	Validation Accuracy
No Regularization	87%
L2 Regularization	90%
Dropout ($p=0.4$)	92%

Mini Batch with dropout is best choice.

b).

$$\text{Leaky ReLU}(a_i) = \begin{cases} z_i & z_i > 0 \\ 0.1 z_i & z_i \le 0 \end{cases}$$

$$z_i = \begin{bmatrix} 3 \\ -4 \\ 2 \end{bmatrix} \qquad \frac{\partial L}{\partial a} = \begin{bmatrix} 5 \\ -1 \\ 4 \end{bmatrix}$$

Local gradient:

$$\frac{\partial a_i}{\partial z_i} = \begin{cases} 1 & z_i > 0 \\ 0.1 & z_i \le 0 \end{cases}$$

$z_1 = 3 > 0 \Rightarrow \frac{\partial a_1}{\partial z_1} = 1$ $\qquad$ $z_3 = 2 > 0 \Rightarrow \frac{\partial a_3}{\partial z_3} = 1$

$z_2 = -4 < 0 \Rightarrow \frac{\partial a_2}{\partial z_2} = 0.1$

$$\text{Jacobian matrix} = J_{LeakyReLU}(z) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Downstream Gradient

$$\frac{\partial L}{\partial z} = \frac{\partial L}{\partial a} \cdot \frac{\partial a}{\partial z} = \begin{bmatrix} 5 \times 1 \\ -1 \times 0.1 \\ 4 \times 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -0.1 \\ 4 \end{bmatrix}$$

c).

$v(\{\}) = 50$ $\quad$ $v(\{c\}) = 60$

$v(\{E\}) = 70$ $\quad$ $v(\{E, c\}) = 90$

Case	Subsets without C	Contribution of C
1.	$\{\}$	$v(\{c\}) - v(\{\}) = 60 - 50 = 10$
2.	$\{E\}$	$v(\{E, c\}) - v(\{E\}) = 90 - 70 = 20$

Shapley value for feature c

$$= \sum_{\underset{\text{subset without } c}{z'}} \frac{\overset{\text{\# features in subset}}{|z'|}!\,(M - |z'| - 1)!}{\underset{\text{total \# feature}}{M!}} \left[v(z' \cup c) - v(z') \right]$$

For case 1, $|z'| = 0$, $M = 2$

$$\frac{|z'|!\,(M - |z'| - 1)!}{M!} = \frac{0!\,(2 - 0 - 1)!}{2!} = \frac{1}{2}$$

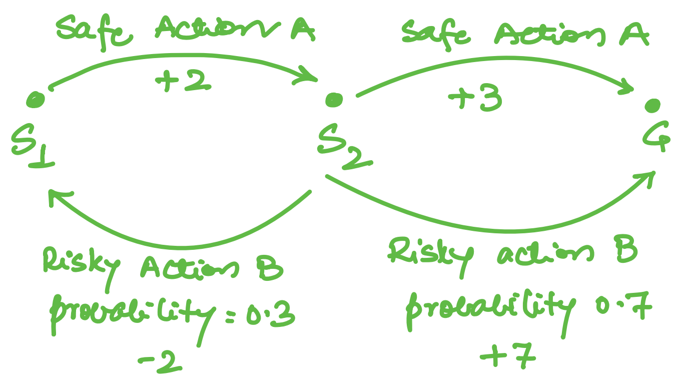
For case 2, $|z'| = 1$, $M = 2$

$$\frac{|z'|!\,(M - |z'| - 1)!}{M!} = \frac{1!\,(2 - 1 - 1)!}{2!} = \frac{1}{2}$$

$$\phi_c = \frac{1}{2} \times 10 + \frac{1}{2} \times 20 = 5 + 10 = 15$$

Ques 5:

a).



$V(G) = 0$

$\gamma = 0.9$

Way 1: If initial $V_0^\pi(G) = 0$ and no info about $V_0^\pi(S_2)$

Policy π always chooses safe (A) in both S_1 and S_2.

$$V_1^\pi(S_2) = 3 + \gamma\, V_0^\pi(G) = 3 + 0.9 * 0 = 3$$

$$V_2^\pi(S_1) = 2 + \gamma\, V_1^\pi(S_2) = 2 + 0.9 * 3 = 4.7$$

Risky (B) in S_2

$$Q^\pi(S_2, B) = \sum_{s'} P(s' \mid S_2, B)\left[r(S_2, B, s') + \gamma\, V^\pi(s')\right]$$

$$= 0.7\left[7 + 0.9 \times 0\right] + 0.3\left[-2 + 0.9 * 4.7\right]$$

$$= 0.7 * 7 + 0.3 * 2.23 = 4.9 + 0.669$$

$$= 5.569$$

Way 2: If initial $V_0^\pi(S_1) = 0$, $V_0^\pi(S_2) = 0$, $V_0^\pi(G) = 0$

$$V_1^\pi(S_1) = 2 + \gamma\, V_0^\pi(S_2) = 2 + 0.9 * 0 = 2$$

$$V_1^\pi(S_2) = 3 + \gamma\, V_0^\pi(G) = 3 + 0.9 * 0 = 3$$

Risky (B) in S_2

$$Q^*(S_2, B) = \sum_{s'} P(s' \mid s_2, B) \left[r(s_2, B, s') + \gamma V^*(s') \right]$$

$$= 0.7 \left[7 + 0.9 \times 0 \right] + 0.3 \left[-2 + 0.9 * 2 \right]$$

$$= 0.7 * 7 + 0.3 * (-0.2) = 4.9 - 0.06$$

$$= 4.84$$

b).

$$P \xrightarrow{+2} T \xrightarrow{+6} D \qquad V(D) = 0$$

Discount factor $\gamma = 0.5$
Learning rate $\alpha = 0.5$

After first episode, $V_1(P) = 1$, $V_1(T) = 3$

$$V_2(P) = V_1(P) + \alpha \left[r + \gamma V_1(T) - V_1(P) \right]$$

$$= 1 + 0.5 \left[2 + 0.5 * 3 - 1 \right]$$

$$= 1 + 0.5 \left[2.5 \right] = 1 + 1.25 = 2.25$$

$$V_2(T) = V_1(T) + \alpha \left[r + \gamma V_1(D) - V_1(T) \right]$$

$$= 3 + 0.5 \left[6 + 0.5 * 0 - 3 \right]$$

$$= 3 + 0.5 \left[3 \right] = 3 + 1.5 = 4.5$$